**Lecture 5**

**1. SECOND-ORDER DIFFERENTIAL EQUATIONS**

***A*** ***second-order*** ***differential equation*** is an equation involving unknown function , first and second order derivatives of an unknown function

 (1)

or

. (1')

A function  is a solution of a differential equation (1) if the equation is satisfied identically when y and its derivatives are substituted into the equation.

**Cauchy problem**. As a rule, it requires two conditions to determine values for two arbitrary constants in the general solution of an second-order differential equation (one condition for each constant)

, . (2)

After finding the unknowns  and  we put they in It gives us a particular solution.

Example. Show that the function  is a solution of the differential equation. Find a particular solution satisfying the given initial conditions , .

Solution. Find first and second order derivatives:

, .

Put them into equation:



 

Function satisfies the given equation. This means that it is a solution of equation.

Satisfying the initial conditions: , , :



So the particular solution is

.

**2. LOWERING THE ORDER**

**OF DIFFERENTIAL EQUATION**

We notice a number of particular cases in which the order of an equation can be lowered.

1. Letthe right-hand sideof differential equation  is just function of *x*:

.

Being that  we rewrite the equation

.

Integration gives us



where  is a arbitrary constant.

It is known that  , so

.

Integration gives us:

,

where  is a arbitrary constant.

**Example.** Solve the differential equation

.

***Solution***.Being that *у*″= we rewrite the equation

 .

After integration, we have

*у*′=

It is known that  , we take

 .

And integration gives us

### ,

where,  and С2 are arbitrary constants.

2. Let the function *у(х*) and a certain number of consecutive derivatives of y: y’, y”, be excluded from the equation, which has the form:

.

We introduce the new variable , thus lowering the order of the equation. Here  is function of , . Then, , . Substituting these values into the equation we get:

.

On finding the general solution of the last equation, , we can find y from the equation

, or .

Solving this equation we get:

.

**Example.** Solve the differential equation .

***Solution***. We denoted , , then equation turn into a first order differential equation:

.

*or*,



сызықты теңдеу аламыз, мұнда, ** , **. Теңдеудің шешімін табу үшін екінші дәрістегі (3) формуланы қолданамыз:



.

Сызықты теңдеу шешімі: .

 болғандықтан,

, .

and integration gives us:

.

Соңында берілген екінші ретті дифференциалдық теңдеу шешімін табамыз:

.

3. If the equation does not contain independent variable , i.e. has the form:

.

We take *y* as independent variable and introduce the new variable , thus lowering the order of the equation. If we take *z* as a function of *y*, and dependent on *x* via *y*, and use the rule for differentiation of a function, we get the following expressions for the derivatives of y with respect to x:

, .

And it is clear from these that the order of the equation is fest in the new variables:

.

On finding the general solution of the last equation, , we can find y from the equation

 or .

Solving this equation we get:

,

where,  and С2 are arbitrary constants.

**Example.** Solve the differential equation .

***Solution***.  деп алсақ, онда:

,

және берілген теңдеу

.

түріне келеді. Бұл айнымалысы ажыратылатын теңдеу:

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Интегралдасақ,

, .

 болғандықтан, , осыдан:



немесе екінші ретті дифференциалдық теңдеу шешімін табамыз:

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